Neglecting higher power & equate the coe. of x.

$b_1 = 1$

$b_2 = a_1 = a_1 b_1$

$b_3 = a_1^2 + a_2 = a_1 b_2 + a_2 b_1$

$b_4 = a_1 b_3 + a_2 b_2 + a_3 b_1$

We have $f(x) = xe^x - 1 = 0$

$$f(x) = x\left[1 + x + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \cdots\right] - 1$$

$$f(x) = \left[x + x^2 + \frac{x^3}{\lfloor 2} + \frac{x^4}{\lfloor 3} + \cdots\right] - 1$$

Then by Ramanujan method we get

$$\left[1 - \left(x + \frac{x^2}{\lfloor 2} + \frac{x^4}{\lfloor 3} + \cdots\right)\right]^{-1} = b_1 + b_2 x + b_3 x^2 + \cdots$$

Now comparing this eq^n with roots of a_1, a_2, a_3 we get

$$\left[1 - (a_1 x + a_2 x^2 + a_3 x^3 + \cdots)\right]^{-1} = b_1 + b_2 x + b_3 x^2 + \cdots$$

$a_1 = 1$, $a_2 = 1$, $a_3 = \frac{1}{\lfloor 2}$, $a_4 = \frac{1}{\lfloor 3}$, $a_5 = \frac{1}{\lfloor 4}$

also $b_1 = 1$

$b_2 = a_1 b_1 = 1 \cdot 1 = 1$

$b_3 = a_1 b_2 + a_2 b_1 = 1 \cdot 1 + 1 \cdot 1 = 2$

$b_4 = a_1 b_3 + a_2 b_2 + a_3 b_1 = 1 \cdot 2 + 1 \cdot 1 + \frac{1}{2} \times 1$

$b_4 = 7/2 = 3.5$

$b_5 = a_1 \cdot b_4 + a_2 b_3 + a_3 b_2 + a_4 b_1$

$= 1 \times 3.5 + 1 \times 2 + \frac{1}{\lfloor 2} \times 1 + \frac{1}{6} \times 1$

$b_5 = 3.5 + 2 + 0.5 + 0.16$

$b_5 = 6.16$

$b_6 = a_1 b_5 + a_2 b_4 + a_3 b_3 + a_4 b_2 + a_5 b_1$

$= 1 \times 6.16 + 1 \times 3.5 + \frac{1}{\lfloor 2} \times 2 + \frac{1}{6} \times 1 + \frac{1}{24} \times 1$

$b_6 = 6.16 + 3.5 + 1 + 0.16 + 0.041$

$b_6 = 10.861$

Therefore $b_1 / b_2 = 1/1 = 1$

$b_2 / b_3 = 1/2 = 0.5$

$b_3 / b_4 = 2/3.5 = 0.571$

$b_4 / b_5 = 3.5/6.16 = 0.568$

$b_5 / b_6 = 6.16/10.861 = 0.5691$

Ques: Find the roots of eq^n $\sin x = 1 - x$

$f(x) = 1 - x - \sin x$

$$= 1 - x - \left(x - \frac{x^3}{\lfloor 3} + \frac{x^5}{\lfloor 5} - \frac{x^7}{\lfloor 7} + \cdots\right)$$

$$f(x) = 1 - 2x + \frac{x^3}{\lfloor 3} - \frac{x^5}{\lfloor 5} + \frac{x^7}{\lfloor 7} + \cdots$$

Then by Ramanujan Method we get–

$$\left[1 - \left(2x - \frac{x^3}{\lfloor 3} + \frac{x^5}{\lfloor 5} - \frac{x^7}{\lfloor 7} + \cdots\right)\right]^{-1}$$

$$= b_1 + b_2 x + b_3 x^2 + \cdots$$

On considering the Rhombous in the table we get

$$\Delta_1^{(1)}$$

$$q_1^{(2)} \qquad q_2^{(1)}$$

$$\Delta_1^{(2)}$$

In this no.s all the component is calculate except $q_2^{(1)}$ which is calculated by using formula

$$\Delta_r^{(i)}\, q_{r+1}^{(i)} = \Delta_r^{(i+1)}\, q_r^{(i+1)}$$

On replacing $i = r = 1$ we get

$$\Delta_1^{(1)}\, q_2^{(1)} = \Delta_1^{(2)}\, q_1^{(2)}$$

$$q_2^{(1)} = \frac{\Delta_1^{(2)}\, q_1^{(2)}}{\Delta_1^{(1)}}$$

$$\rightarrow \quad q_2^{(1)} = \frac{-0.56 \times 4.16}{-1.84}$$

$$q_2^{(1)} = 1.266$$

Illy on considering the another rhombous from the table we get

$$q_2^{(0)}$$

$$\Delta_1^{(1)} \qquad \Delta_2^{(0)}$$

$$q_2^{(1)}$$

In this all the quantities are known except $\Delta_2^{(0)}$ whose value can be calculated by considering the formula.

$$\Delta_r^{(i)} + q_r^{(i)} = \Delta_{r-1}^{(i+1)} + q_r^{(i+1)}$$

On replacing $r = 2$ & $i = 0$ we get

$$\Delta_2^{(0)} + q_2^{(0)} = \Delta_1^{(1)} + q_2^{(1)}$$

$$\Delta_2^{(0)} = -1.84 + 1.266 \qquad q_2^{(0)} = 0$$

$$\Delta_2^{(0)} = -0.54 \text{ Ans}$$

also the component of $\Delta_3^{(-1)} = 0$

Now the next another rhombous from table

$$\Delta_2^{(0)}$$

$$q_2^{(1)} \qquad q_3^{(0)}$$

$$\Delta_2^{(1)}$$

In this all the quantities are known except $\Delta_2^{(1)}$ whose value can be calculated by the formula

$$\Delta_r^{(i)} + q_r^{(i)} = \Delta_{r-1}^{(i+1)} + q_r^{(i+1)}$$

On replacing $r = 2$ & $i = 1$ we get

$$\Delta_2^{(1)} + q_2^{(1)} = \Delta_1^{(2)} + q_2^{(2)}$$

Q7 Ramanjuan Method of Itteration Shrniwash Ramanujan gave a itteration method to give the proper root of algebric eq^n $f(x)=0$ If any func^n is expressed in the form of

$$f(x) = a_1 x + a_2 x^2 + a_3 x^3 + --$$

Then the another component $b_1, b_2, -- b_4 --$ are related with these components whose rel^n can be expressed as

$$[1-(a_1 x + a_2 x^2 + a_3 x^3 + --)]^{-1} = b_1 + b_2 x + b_3 x^2 + --$$

where the roots b_1, b_2, b_3 give the approximate roots of algebric eq^n to the ratio of these roots comes out

Then the unit term represent the proper roots of algebric eq^n i.e the b_n / b_{n-1} give the unit value then the roots $(b_n \& b_{n+1})$ represent the ~~[illegible]~~ root of algebric eq^n $f(x)=0$

Ques find the smallest roots of the eq^n

$$f(x) = x^3 - 6x^2 + 11x - 6 = 0$$

Sol^n

from Ramajunn method we know that

$$[1-(a_1 x + a_2 x^2 + a_3 x^3 + --)]^{-1} = b_1 + b_2 x + b_3 x^2 + --$$

Then by binomial theorem we get

$$1 + (a_1 x + a_2 x^2 + a_3 x^3 + --) + \frac{-1(-1-1)}{\lfloor 2}(a_1 x + a_2 x^2 +$$

$$a_3 x^3 + --)^2 + -- = b_1 + b_2 x^1 + b_3 x^2 + --$$

On neglecting higher powers we get

$$1 + (a_1 x + a_2 x^2 + a_3 x^3 + --) + (a_1 x + a_2 x^2 + a_3 x^3 + --)^2$$
$$= b_1 + b_2 x + b_3 x^2 + --$$

Now on comparing component of x

$$b_1 = 1$$
$$b_2 = a_1 = a_1 b_1$$
$$b_3 = a_1^2 + a_2 = a_1 b_2 + a_2 b_1$$
$$b_4 = a_1 b_3 + a_2 b_2 + a_3 b_1$$
$$\vdots$$
$$b_n = a_1 b_{n-1} + a_2 b_{n-2} + -- a_{n-1} b_1$$

we have $f(x) = x^3 - 6x^2 + 11x - 6 = 0$

Then by Ramanujan method

$$\left[1 - \left(\frac{11x - 6x^2 + x^3}{6}\right)\right]^{-1} = b_1 + b_2 x + b_3 x^2 + --$$

Now on comparing this eq^n with roots of a_1, a_2, a_3 --

$$a_1 = \frac{11}{6}$$

$$\left[1 - (a_1 x + a_2 x^2 + a_3 x^3 + --)\right]^{-1} = b_1 + b_2 x + b_3 x^2 + --$$

$a_1 = \frac{11}{6}$, $a_2 = -1$, $a_3 = \frac{1}{6}$, $a_4 = a_5 = \ldots 0$

also $b_1 = 1$

$b_2 = 11/6$

$b_3 = \frac{121}{36} - 1 = \frac{85}{36}$

$b_4 = a_1 b_3 + a_2 b_2 + a_3 b_1 = \frac{11}{6} \times \frac{85}{36} + \frac{11}{6} + \frac{1}{6} \times 1$

$= \frac{11 \times 85 - 36 \times 10}{36 \times 6}$

$= \frac{935 - 360}{36 \times 6} = \frac{575}{216}$

$b_5 = a_1 b_4 + a_2 b_3 + a_3 b_2 + a_4 b_1$

$= \frac{11}{6} \times \frac{575}{216} + \frac{85}{36} + \frac{1}{6} \times \frac{11}{6}$

$= \frac{11 \times 575 - 74 \times 36 +}{216 \times 6}$

$= \frac{6325 - 2664}{1296}$

$\rightarrow b_5 = \frac{3661}{1296}$

$\rightarrow b_6 = a_1 b_5 + a_2 b_4 + a_3 b_3 + a_4 b_2 + a_5 b_1$

$b_6 = \frac{11}{6} \times \frac{3661}{1296} + \frac{575}{216} + \frac{1}{6} \times \frac{85}{36}$

$b_6 = \frac{22631}{7776}$

Therefore $b_1/b_2 = 6/11 = 54545$

Illy $b_2/b_3 = \frac{11/6}{85/36} = \frac{66}{85} = 0.7764$

$b_3/b_4 = \frac{85/36}{575/216} = \frac{510}{575} = 0.8867$

$b_4/b_5 = \frac{575/216}{3661/1296} = \frac{3450}{3661} = 0.9423$

$b_5/b_6 = \frac{3661/1296}{2263/7776} = \frac{3138}{3233} = 0.9706$

These are the smallest roots of above eqn

Ques. Find the roots of $f(x) = xe^x = 1$

Soln from Ramanujan method we know that $[1 - (a_1 x + a_2 x^2 + a_3 x^3 + \ldots)]^{-1} = b_1 + b_2 x + b_3 x^2 + \ldots$

Then by binomial theorem we get

$1 + (a_1 x + a_2 x + a_3 x^3 + \ldots) + (a_1 x + a_2 x^2 + a_3 x^3 + \ldots)^2$
$= b_1 + b_2 x + b_3 x^2 + \ldots$